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Constant-Cutoff Approach to Meson–Hyperon Couplings

Nils Dalarsson[1]

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The kaon coupling constants at hyperon–nucleon vertices and the pion coupling constants at hyperon–hyperon vertices are calculated in the framework of the constant-cutoff approach to the CHK bound-state model of hyperons, where the positive-parity hyperons such as Λ, Σ, and $\Sigma^* = \Sigma(1385)$ are the P-wave bound states of an antikaon and the $SU(2)$ Skyrme soliton, while $\Lambda^* = \Lambda(1405)$ is the S-wave bound state. Meson coupling constants are defined as matrix elements of the meson-source terms between two single-baryon states following the method developed for resolving the Yukawa coupling problem in the $SU(2)$ Skyrme soliton model. The magnitudes of the meson coupling constants are found to be close to those obtained using the complete Skyrme model and the phenomenological values.

1. INTRODUCTION

It was shown by Skyrme (1961, 1962) that baryons can be treated as solitons of a nonlinear chiral theory. The original Lagrangian of the chiral $SU(2)$ σ-model is

$$\mathcal{L} = \frac{F_\pi^2}{16} \operatorname{Tr} \partial_\mu U \, \partial^\mu U^+ \tag{1.1}$$

where

$$U = \frac{2}{F_\pi} (\sigma + i\boldsymbol{\tau} \cdot \boldsymbol{\pi}) \tag{1.2}$$

is a unitary operator ($UU^+ = 1$) and F_π is the pion-decay constant. In (1.2) $\sigma = \sigma(\mathbf{r})$ is a scalar meson field and $\boldsymbol{\pi} = \boldsymbol{\pi}(\mathbf{r})$ is the pion-isotriplet.

[1] Royal Institute of Technology, Stockholm, Sweden.

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The classical stability of the soliton solution to the chiral σ-model Lagrangian requires the additional ad hoc term, proposed by Skyrme (1961), to be added to (1.1)

$$\mathcal{L}_{\mathrm{Sk}} = \frac{1}{32e^2}\,\mathrm{Tr}[U^+\partial_\mu U,\ U^+\partial_\nu U]^2 \tag{1.3}$$

with a dimensionless parameter e and where $[A, B] = AB - BA$. It was shown by several authors (Adkins *et al.*, 1983; Adkins and Nappi, 1984; for an extensive list of other references see Holzwarth and Schwesinger, 1986; Nyman and Riska, 1990) that, after the collective quantization using the spherically symmetric ansatz

$$U_0(\mathbf{r}) = \exp[i\boldsymbol{\tau}\cdot\hat{\mathbf{r}}F(r)], \qquad \hat{\mathbf{r}} = \mathbf{r}/r \tag{1.4}$$

the chiral model, with both (1.1) and (1.3) included, gives good agreement with experiment for several important physical quantities. Thus it should be possible to derive the effective chiral Lagrangian, obtained as a sum of (1.1) and (1.3), from a more fundamental theory like QCD. On the other hand it is not easy to generate a term like (1.3) and give a clear physical meaning to the dimensionless constant e in (1.3) using QCD.

Mignaco and Wulck (1989) (MW) indicated therefore the possibility to build a stable single-baryon ($n = 1$) quantum state in the simple chiral theory with the Skyrme stabilizing term (1.3) omitted. MW showed that the chiral angle $F(r)$ is in fact a function of a dimensionless variable $s = \frac{1}{2}\chi''(0)r$, where $\chi''(0)$ is an arbitrary dimensional parameter intimately connected to the usual stability argument against the soliton solution for the nonlinear σ-model Lagrangian.

Using the adiabatically rotated ansatz $U(\mathbf{r}, t) = A(t)U_0(\mathbf{r})A^+(t)$, where $U_0(\mathbf{r})$ is given by (1.4), MW obtained the total energy of the nonlinear σ-model soliton in the form

$$E = \frac{\pi}{4}F_\pi^2\frac{1}{\chi''(0)}a + \frac{1}{2}\frac{[\chi''(0)]^3}{\frac{1}{4}\pi F_\pi^2 b}J(J+1) \tag{1.5}$$

where

$$a = \int_0^\infty \left[\frac{1}{4}s^2\left(\frac{d\mathcal{F}}{ds}\right)^2 + 8\sin^2\left(\frac{1}{4}\mathcal{F}\right)\right]dr \tag{1.6}$$

$$b = \int_0^\infty ds\,\frac{64}{3}s^2\sin^2\left(\frac{1}{4}\mathcal{F}\right) \tag{1.7}$$

and $\mathcal{F}(s)$ is defined by

$$F(r) = F(s) = -n\pi + \tfrac{1}{4}\mathcal{F}(s) \tag{1.8}$$

The stable minimum of the function (1.5) with respect to the arbitrary dimensional scale parameter $\chi''(0)$ is

$$E = \frac{4}{3} F_\pi \left[\frac{3}{2} \left(\frac{\pi}{4} \right)^2 \frac{a^3}{b} J(J + 1) \right]^{1/4} \tag{1.9}$$

Despite the nonexistence of the stable classical soliton solution to the nonlinear σ-model, it is possible, after the collective coordinate quantization, to build a stable chiral soliton at the quantum level, provided that there is a solution $F = F(r)$ which satisfies the soliton boundary conditions, i.e., $F(0) = -n\pi$, $F(\infty) = 0$, such that the integrals (1.6) and (1.7) exist.

However, as pointed out by Iwasaki and Ohyama (1989), the quantum stabilization method in the form proposed by MW is not correct, since in the simple σ-model the conditions $F(0) = -n\pi$ and $F(\infty) = 0$ cannot be satisfied simultaneously. In other words, if the condition $F(0) = -\pi$ is satisfied, Iwasaki and Ohyama obtained numerically $F(\infty) \to -\pi/2$, and the chiral phase $F = F(r)$ with correct boundary conditions does not exist.

Iwasaki and Ohyama also proved analytically that both boundary conditions $F(0) = -n\pi$ and $F(\infty) = 0$ cannot be satisfied simultaneously. Introducing a new variable $y = 1/r$ into the differential equation for the chiral angle $F = F(r)$, we obtain

$$\frac{d^2F}{dy^2} = \frac{1}{y^2} \sin 2F \tag{1.10}$$

There are two kinds of asymptotic solutions to equation (1.10) around the point $y = 0$, which is called a regular singular point if $\sin 2F \approx 2F$. These solutions are

$$F(y) = \frac{m\pi}{2} + cy^2, \qquad m = \text{even integer} \tag{1.11}$$

$$F(y) = \frac{m\pi}{2} + \sqrt{cy} \cos\left[\frac{\sqrt{7}}{2} \ln(cy) + \alpha \right], \qquad m = \text{odd integer} \tag{1.12}$$

where c is an arbitrary constant and α is a constant to be chosen appropriately. When $F(0) = -n\pi$ then we want to know which of these two solutions are approached by $F(y)$ when $y \to 0$ ($r \to \infty$). In order to answer that question we multiply (1.10) by $y^2F'(y)$, integrate with respect to y from y to ∞, and use $F(0) = -n\pi$. Thus we get

$$y^2F'(y) + \int_y^\infty 2y[F'(y)]^2\, dy = 1 - \cos[2F(y)] \tag{1.13}$$

Since the left-hand side of (1.13) is always positive, the value of $F(y)$ is always limited to the interval $n\pi - \pi < F(y) < n\pi + \pi$. Taking the limit $y \to 0$, we find that (1.13) is reduced to

$$\int_0^\infty 2y[F'(y)]^2\, dy = 1 - (-1)^m \tag{1.14}$$

where we used (1.11)–(1.12). Since the left-hand side of (1.14) is strictly positive, we must choose an odd integer m. Thus the solution satisfying $F(0) = -n\pi$ approaches (1.12) and we have $F(\infty) \neq 0$. The behavior of the solution (1.11) in the asymptotic region $y \to \infty$ ($r \to 0$) is investigated by multiplying (1.10) by $F'(y)$, integrating from 0 to y, and using (1.11). The result is

$$[F'(y)]^2 = \frac{2\sin^2 F(y)}{y^2} + \int_0^y \frac{2\sin^2 F(y)}{y^3}\, dy \tag{1.15}$$

From (1.15) we see that $F'(y) \to$ const as $y \to \infty$, which means that $F(r) \simeq 1/r$ for $r \to 0$. This solution has a singularity at the origin and cannot satisfy the usual boundary condition $F(0) = -n\pi$.

In Dalarsson (1991a,b, 1992), I suggested a method to resolve this difficulty by introducing a radial modification phase $\varphi = \varphi(r)$ in the ansatz (1.4) as follows:

$$U(\mathbf{r}) = \exp[i\boldsymbol{\tau}\cdot\mathbf{r}_0 F(r) + i\varphi(r)], \qquad \mathbf{r}_0 = \mathbf{r}/r \tag{1.16}$$

Such a method provides a stable chiral quantum soliton, but the resulting model is an entirely noncovariant chiral model, different from the original chiral σ-model.

In the present paper we use the constant-cutoff limit of the cutoff quantization method developed by Balakrishna *et al.* (1991) (see also Jain *et al.*, 1989) to construct a stable chiral quantum soliton within the original chiral σ-model. Then we apply this method to calculate the kaon coupling constants at hyperon–nucleon vertices and the pion coupling constants at hyperon–hyperon vertices in the framework of the constant-cutoff approach to the CHK bound-state model of hyperons, where the positive-parity hyperons such as Λ, Σ, and $\Sigma^* = \Sigma(1385)$ are the P-wave bound states of an antikaon and the $SU(2)$ Skyrme soliton, while $\Lambda^* = \Lambda(1405)$ is the S-wave bound state. Meson coupling constants are defined as matrix elements of the meson-source terms between two single-baryon states following the method developed for resolving the Yukawa coupling problem in the $SU(2)$ Skyrme soliton model (Hayashi *et al.*, 1992; Saito and Uehara, 1995). The magnitudes of the meson coupling constants are found to be close to those obtained using the complete Skyrme model (Kondo *et al.*, 1996) and the phenomenological values (Lee *et al.*, 1994, 1995).

The reason why the cutoff approach to the problem of the chiral quantum soliton works is connected to the fact that the solution $F = F(r)$ which satisfies the boundary condition $F(\infty) = 0$ is singular at $r = 0$. From the physical point of view the chiral quantum model is not applicable to the region about the origin, since in that region there is a quark-dominated bag of the soliton.

However, as argued in Balakrishna *et al.* (1991), when a cutoff ϵ is introduced, then the boundary conditions $F(\epsilon) = -n\pi$ and $F(\infty) = 0$ can be satisfied. In Balakrishna *et al.* (1991) an interesting analogy with the damped pendulum is discussed, showing clearly that as long as $\epsilon > 0$, there is a chiral phase $F = F(r)$ satisfying the above boundary conditions. The asymptotic forms of such a solution are given by Eq. (2.2) in Balakrishna *et al.* (1991). From these asymptotic solutions we immediately see that for $\epsilon \to 0$ the chiral phase diverges at the lower limit.

Different applications of the constant-cutoff approach have been discussed in Dalarsson (1993, 1995a–d, 1996a–c, 1997).

2. CONSTANT-CUTOFF STABILIZATION

The chiral soliton with baryon number $n = 1$ is given by (1.4), where $F = F(r)$ is the radial chiral phase function satisfying the boundary conditions $F(0) = -\pi$ and $F(\infty) = 0$.

Substituting (1.4) into (1.1), we obtain the static energy of the chiral baryon

$$M = \frac{\pi}{2} F_\pi^2 \int_{\epsilon(t)}^{\infty} dr \left[r^2 \left(\frac{dF}{dr}\right)^2 + 2 \sin^2 F \right] \tag{2.1}$$

In (2.1) we avoid the singularity of the profile function $F = F(r)$ at the origin by introducing the cutoff $\epsilon(t)$ at the lower boundary of the space interval $r \in [0, \infty]$, i.e., by working with the interval $r \in [\epsilon, \infty]$. The cutoff itself is introduced, following Balakrishna *et al.* (1991), as a dynamic time-dependent variable.

From (2.1) we obtain the following differential equation for the profile function $F = F(r)$:

$$\frac{d}{dr}\left(r^2 \frac{dF}{dr}\right) = \sin 2F \tag{2.2}$$

with the boundary conditions $F(\epsilon) = -\pi$ and $F(\infty) = 0$, such that the correct soliton number is obtained. The profile function $F = F[r; \epsilon(t)]$ now depends implicitly on time t through $\epsilon(t)$. Thus in the nonlinear σ-model Lagrangian

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\partial_\mu U \, \partial^\mu U^+) \, d^3x \tag{2.3}$$

we use the ansätze

$$U(\mathbf{r}, t) = A(t)U_0(\mathbf{r}, t)A^+(t), \qquad U^+(\mathbf{r}, t) = A(t)U_0^+(\mathbf{r}, t)A^+(t) \tag{2.4}$$

where

$$U_0(\mathbf{r}, t) = \exp\{i\boldsymbol{\tau}\cdot\mathbf{r}_0 F[r; \epsilon(t)]\} \tag{2.5}$$

The static part of the Lagrangian (2.3), i.e.,

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\nabla U \cdot \nabla U^+) \, d^3x = -M \tag{2.6}$$

is equal to minus the energy M given by (2.1). The kinetic part of the Lagrangian is obtained using (2.4) with (2.5) and it is equal to

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\partial_0 U \, \partial_0 U^+) \, d^3x = bx^2 \, \mathrm{Tr}[\partial_0 A \, \partial_0 A^+] + c[\dot{x}(t)]^2 \tag{2.7}$$

where

$$b = \frac{2\pi}{3} F_\pi^2 \int_1^\infty \sin^2 F \, y^2 \, dy, \qquad c = \frac{2\pi}{9} F_\pi^2 \int_1^\infty y^2 \left(\frac{dF}{dy}\right)^2 y^2 \, dy \tag{2.8}$$

with $x(t) = [\epsilon(t)]^{3/2}$ and $y = r/\epsilon$. On the other hand, the static energy functional (2.1) can be rewritten as

$$M = ax^{2/3}, \qquad a = \frac{\pi}{2} F_\pi^2 \int_1^\infty \left[y^2 \left(\frac{dF}{dy}\right)^2 + 2 \sin^2 F \right] dy \tag{2.9}$$

Thus the total Lagrangian of the rotating soliton is given by

$$L = c\dot{x}^2 - ax^{2/3} + 2bx^2 \dot{\alpha}_\nu \dot{\alpha}^\nu \tag{2.10}$$

where $\mathrm{Tr}(\partial_0 A \ \partial_0 A^+) = 2\dot{\alpha}_\nu \dot{\alpha}^\nu$ and α_ν ($\nu = 0, 1, 2, 3$) are the collective coordinates defined as in Bhaduri (1988). In the limit of a time-independent cutoff ($\dot{x} \to 0$) we can write

$$H = \frac{\partial L}{\partial \dot{\alpha}^\nu} \dot{\alpha}^\nu - L = ax^{2/3} + 2bx^2 \dot{\alpha}_\nu \dot{\alpha}^\nu = ax^{2/3} + \frac{1}{2bx^2} J(J+1) \tag{2.11}$$

where $\langle \mathbf{J}^2 \rangle = J(J+1)$ is the eigenvalue of the square of the soliton laboratory angular momentum. A minimum of (2.11) with respect to the parameter x is reached at

$$x = \left[\frac{2}{3}\frac{ab}{J(J+1)}\right]^{-3/8} \Rightarrow \epsilon^{-1} = \left[\frac{2}{3}\frac{ab}{J(J+1)}\right]^{1/4} \tag{2.12}$$

The energy obtained by substituting (2.12) into (2.11) is given by

$$E = \frac{4}{3}\left[\frac{3}{2}\frac{a^3}{b}J(J+1)\right]^{1/4} \tag{2.13}$$

This result is identical to the result obtained by Mignaco and Wulck, which is easily seen if we rescale the integrals a and b in such a way that $a \to (\pi/4)F_\pi^2 a$ and $b \to (\pi/4)F_\pi^2 b$ and introduce $f_\pi = 2^{-3/2}F_\pi$. However, in the present approach, as shown in Balakrishna *et al.* (1991), there is a profile function $F = F(y)$ with proper soliton boundary conditions $F(1) = -\pi$ and $F(\infty) = 0$ and the integrals a, b, and c in (2.9)–(2.10) exist and are shown in Balakrishna *et al.* (1991) to be $a = 0.78$ GeV^2, $b = 0.91$ GeV^2, and $c = 1.46$ GeV^2 for $F_\pi = 186$ MeV.

Using (2.13), we obtain the same prediction for the mass ratio of the lowest states as Mignaco and Wulck (1989), which agrees rather well with the empirical mass ratio for the Δ-resonance and the nucleon. Furthermore, using the calculated values for the integrals a and b, we obtain the nucleon mass $M(N) = 1167$ MeV, which is about 25% higher than the empirical value of 939 MeV. However, if we choose the pion-decay constant equal to $F_\pi = 150$ MeV, we obtain $a = 0.507$ GeV^2 and $b = 0.592$ GeV^2, giving exact agreement with the empirical nucleon mass.

Finally, it is of interest to know how large the constant cutoffs are for the above values of the pion-decay constant in order to check if they are in the physically acceptable ballpark. Using (2.12), it is easily shown that for the nucleons ($J = 1/2$) the cutoffs are equal to

$$\epsilon = \begin{cases} 0.22 \text{ fm} & \text{for} \quad F_\pi = 186 \text{ MeV} \\ 0.27 \text{ fm} & \text{for} \quad F_\pi = 150 \text{ MeV} \end{cases} \tag{2.14}$$

From (2.14) we see that the cutoffs are too small to agree with the size of the nucleon (0.72 fm), as we should expect, since the cutoffs rather indicates the size of the quark-dominated bag in the center of the nucleon. Thus we find that the cutoffs are of reasonable physical size. Since the cutoff is proportional to F_π^{-1}, we see that the pion-decay constant must be less than 57 MeV in order to obtain a cutoff which exceeds the size of the nucleon. Such values of pion-decay constant are not relevant to any physical phenomena.

3. CHK LAGRANGIAN AND HAMILTONIAN IN THE CONSTANT-CUTOFF MODEL

Callan and Klebanov (1985) showed that a good description of the hyperon spectrum in the Skyrme model (Skyrme, 1961, 1962) is obtained if the hyperons are treated as bound kaon–soliton systems. Callan *et al.* (1988) (CHK) successfully completed this program. The basic idea of their model is to treat strangeness separately from isospin in the Skyrme model, assuming that the vacuum is approximately $SU(3)$-symmetric, i.e., $F_K \approx F_\pi$. The strange baryons are generated by binding kaons in the field of "rotating" $SU(2)$ solitons. Since there is no static field associated with the strangeness number, it is essential in this picture that there exist bound states in the kaon–soliton complex giving rise to hyperons. CHK showed that such bound states exist. A remarkable property of the kaons in this model is that after quantization they look like s-quarks, due to topological effects. This leads to a spectroscopy of hyperons quite similar to that of quark models.

In the CHK approach the kaon–soliton field is written in the form

$$U = \sqrt{U_\pi} U_K \sqrt{U_\pi} \tag{3.1}$$

where U_π is the $SU(3)$-extension of the usual $SU(2)$ skyrmion field used to describe the nucleon spectrum, and U_K is the field describing the kaons:

$$U_\pi = \begin{bmatrix} u_\pi & 0 \\ 0 & 1 \end{bmatrix}, \qquad U_K = \exp\left\{ i \frac{2^{3/2}}{F_\pi} \begin{bmatrix} 0 & K \\ K^+ & 0 \end{bmatrix} \right\} \tag{3.2}$$

The Lagrangian density for a bound kaon–soliton system in the simplified Skyrme model, with the Skyrme stabilizing term (1.3) omitted, is given by

$$\begin{aligned} \mathcal{L}_{\mathrm{CK}} = \frac{F_\pi^2}{16} \operatorname{Tr} \partial_\mu U \, \partial^\mu U^+ + \frac{F_\pi^2}{48} (m_\pi^2 + 2m_{\mathrm{K}}^2) \operatorname{Tr}(U + U^+ - 2) \\ + \frac{\sqrt{3}}{24} F_\pi^2 (m_\pi^2 - m_{\mathrm{K}}^2) \operatorname{Tr} \lambda_8 (U + U^+) \end{aligned} \tag{3.3}$$

where m_π and m_{K} are pion and kaon masses, respectively. In (3.2) u_π is the usual $SU(2)$-skyrmion field, given by (1.4), and $F = F(r)$ is a radial function which, for $m_\pi = 0$, satisfies the differential equation (2.2). The two-dimensional vector K in (3.2) is the kaon doublet

$$K = \begin{bmatrix} K^+ \\ K^0 \end{bmatrix}, \qquad K^+ = [K^- \quad \overline{K}^0] \tag{3.4}$$

In addition to the simplified Skyrme-model action obtained using the Lagrangian density (3.3), the Wess–Zumino action in the form

$$S_{\mathrm{WZ}} = -\frac{iN_c}{240\pi^2}\int d^5x\, e^{\mu\nu\alpha\beta\gamma}\,\mathrm{Tr}[U^+\partial_\mu U\; U^+\partial_\nu U\; U^+\partial_\alpha U\; U^+\partial_\beta U\; U^+\partial_\gamma U] \tag{3.5}$$

must be included in the total action of a kaon–soliton system. In (3.5), N_c is the number of colors in the underlying QCD. Using (3.3) and (3.5), we may write

$$\mathcal{L} = \mathcal{L}_{\mathrm{CK}} + \mathcal{L}_{\mathrm{WZ}} = \mathcal{L}_{\mathrm{Sky}} + \mathcal{L}_{\mathrm{K}} + \mathcal{O}(K^3) \tag{3.6}$$

Using (1.2), we obtain

$$\mathcal{L}_{\mathrm{Sky}} = \tfrac{1}{2}\dot{\pi}_i G_{ij}\dot{\pi}_j + \mathcal{M}(\boldsymbol{\pi}, \partial_k\boldsymbol{\pi}) \tag{3.7}$$

where $\mathcal{M}(\boldsymbol{\pi}, \partial_K\boldsymbol{\pi})$ is easily obtained from (3.3) and the metric G_{ij} is given by

$$G_{ij} = \delta_{ij} + \frac{\pi_i\pi_j}{\sigma^2} = \delta_{ij} + \tan^2 F\,\hat{r}_i\hat{r}_j \tag{3.8}$$

Furthermore, $\mathcal{L}_K$ is a bilinear in K^+ and K, given by

$$\begin{aligned}\mathcal{L}_K = {} & (D_\mu K)^+(D^\mu K) - m_K^2 K^+K + \frac{m_\pi^2}{2}\left(1 - \frac{2\sigma}{F_\pi}\right)K^+K \\ & - \frac{1}{F_\pi^2}\,\partial_\mu\pi_k G_{km}\partial^\mu\pi_m\, K^+K - i\frac{N_c}{F_\pi^2}\,B_\mu[K^+(D^\mu K) - (D^\mu K)^+K]\end{aligned} \tag{3.9}$$

where

$$D_\mu = \partial_\mu + \mathbf{V}\cdot\partial_\mu\boldsymbol{\pi} \tag{3.10}$$

$$\mathbf{V} = \frac{2i}{F_\pi^2}\,\frac{1}{1 + 2\sigma/F_\pi}\,\boldsymbol{\tau}\times\boldsymbol{\pi} \tag{3.11}$$

$$B_\mu = \frac{1}{24\pi^2}\,\epsilon_{\mu\nu\alpha\beta}\,\mathrm{Tr}(u_\pi^+\partial^\nu u_\pi\; u_\pi^+\partial^\alpha u_\pi\; u_\pi^+\partial^\beta u_\pi) \tag{3.12}$$

In order to obtain the expression for the Hamiltonian corresponding to the Lagrangian (3.9), we rewrite (3.9) as follows:

$$\mathcal{L}_K = \dot{K}^+\dot{K} + \dot{K}^+(\mathbf{V}\cdot\dot{\boldsymbol{\pi}} + i\lambda)K + K^+(\dot{\boldsymbol{\pi}}\cdot\mathbf{V}^+ - i\lambda)\dot{K} - i\mathbf{X}\cdot\dot{\boldsymbol{\pi}} + \mathcal{L}_0 \tag{3.13}$$

where $\mathcal{L}_0$ does not include any derivatives of kaon or pion fields, and we have

$$\lambda(r) = -\frac{N_c}{2\pi^2 F_\pi^2}\frac{\sin^2 F}{r^2}\frac{dF}{dr} \tag{3.14}$$

$$\mathbf{X} = 2K^+[\lambda\mathbf{V} - \boldsymbol{\lambda}^j(\mathbf{V}\cdot\partial_j\boldsymbol{\pi})]K + \boldsymbol{\lambda}^j(\partial_j K^+ K - K^+\partial_j K) \tag{3.15}$$

In (3.15) $\boldsymbol{\lambda}^j$ is defined using

$$\boldsymbol{\lambda}^j\cdot\dot{\boldsymbol{\pi}} = \frac{N_c}{F_\pi^2}B^j \tag{3.16}$$

From (3.13) it is now possible to derive the expression for the total Hamiltonian, given by

$$\mathcal{H} = \mathcal{H}_{\mathrm{Sky}} + \mathcal{H}_{\mathrm{K}} + \mathcal{H}_{\pi K} \tag{3.17}$$

where

$$\mathcal{H}_{\mathrm{Sky}} = \frac{1}{2}\dot{P}_i G_{ij}^{-1}\dot{P}_j + \mathcal{M}(\boldsymbol{\pi}, \partial_k\boldsymbol{\pi}) \tag{3.18}$$

$$\begin{aligned}\mathcal{H}_K = {} & (\Pi^+ + i\lambda K^+)(\Pi - i\lambda K) + (D_jK)^+(D^jK)\\ & + \left[m_K^2 - \frac{m_\pi^2}{2}\left(1 - \frac{2\sigma}{F_\pi}\right) - \frac{1}{F_\pi^2}\partial_j\pi_a G_{ab}\partial^j\pi_b\right]K^+K\end{aligned} \tag{3.19}$$

$$\mathcal{H}_{\pi K} = iP_a G_{ab}^{-1}X_b - [(P_a G_{ab}^{-1}V_b K)^+(\Pi - i\lambda K) + \text{h.c.}] \tag{3.20}$$

with all fields defined in the laboratory system. The momentum fields $\mathbf{P}$ and Π, conjugate to $\boldsymbol{\pi}$ and K^+, respectively, are given by

$$P_a = \frac{\delta\mathcal{L}}{\delta\dot{\pi}_a(x)} = G_{ab}\dot{\pi}_b - iX_a + \dot{K}^+V_aK + K^+V_a^+\dot{K} \tag{3.21}$$

$$\Pi = \frac{\delta\mathcal{L}}{\delta\dot{K}^+(x)} = \dot{K} + i\lambda K + \mathbf{V}\cdot\dot{\boldsymbol{\pi}}K \tag{3.22}$$

Since the massive kaon fields do not contain the zero-mode, wave functions of the $SU(2)$ skyrmion and pion fields are the total fields the canonical commutation relations between fields $\boldsymbol{\pi}$ and K^+ and their respective momentum fields $\mathbf{P}$ and Π are

$$[\pi_a(\mathbf{x}, t), P_b(\mathbf{y}, t)]i\,\delta_{ab}\delta(\mathbf{x} - \mathbf{y}) \tag{3.23}$$

$$[K_\alpha^+(\mathbf{x}, t), \Pi_\beta(\mathbf{y}, t)]i\,\delta_{\alpha\beta}\delta(\mathbf{x} - \mathbf{y}) \tag{3.24}$$

In order to calculate the kaon scattering amplitudes, we introduce the asymptotic kaon fields

$$K_{\alpha\mathrm{IN}}(x) = \sum_{\mathbf{k}} \frac{1}{(2\pi)^{3/2}\sqrt{2\omega_k}} [b_\alpha(\mathbf{k})e^{-ikx} + a_\alpha^+(\mathbf{k})e^{ikx}] \tag{3.25}$$

$$K_{\alpha\mathrm{IN}}^+(x) = \sum_{\mathbf{k}} \frac{1}{(2\pi)^{3/2}\sqrt{2\omega_k}} [a_\alpha(\mathbf{k})e^{-ikx} + b_\alpha^+(\mathbf{k})e^{ikx}] \tag{3.26}$$

with $\omega_k = \sqrt{\mathbf{k}^2 + m_K^2}$. In (3.26) $a_\alpha(\mathbf{k})$ and $b_\alpha(\mathbf{k})$ are the annihilation operators of the antikaon and kaon fields of the in-state with isospin $\alpha = 1/2$ and $-1/2$, respectively. The same forms are introduced for the out-states. The field $K_\alpha(x)$ above is the interpolating field from the in-state to the out-state. The single-baryon state with definite spin, isospin, and momentum is described as the rotated and translated Skyrme soliton with a bound-state antikaon if the baryon carries strangeness. The Fock space is spanned by the in- and out-states composed of the in- and out-creation operators of the mesons acting on the single-baryon states.

Following Kondo (1996), we also note that the single-baryon state is not an eigenstate of the Hamiltonian

$$H = \int d^3\mathbf{x}\, \mathcal{H} = H_{\mathrm{Sky}} + H_K + H_{\pi K} \tag{3.28}$$

but we have

$$\langle B(\mathbf{p}) | H | B(\mathbf{q}) \rangle = E_B(\mathbf{p})\, \delta(\mathbf{p} - \mathbf{q}) \tag{3.29}$$

with (Kondo, 1996)

$$E_B = M_B + \mathbf{p}^2/(2M_B) + \mathcal{O}(N_c^{-2}) \tag{3.30}$$

4. KAON AND PION COUPLINGS TO POSITIVE-PARITY HYPERONS

4.1. Kaon Couplings

Using the LSZ reduction formula, we obtain the scattering amplitude for the process $\overline{K}_\alpha(\mathbf{k}) + N(\mathbf{p}) \to \overline{K}_\beta(\boldsymbol{\kappa}) + N(\mathbf{q})$:

$$\begin{aligned} T_{\overline{K}N\to\overline{K}N} = i(2\pi)^3 \int d^4x\, e^{i\kappa x} \langle N(\mathbf{q}) | T(J_\beta^{K^+}(x) J_\alpha^K(0)) \\ + \delta(x^0)[\dot{K}_\beta^+(x), J_\alpha^K(0)] - i\omega_\kappa \delta(x^0)[K_\beta^+(x), J_\alpha^K(0)] | N(\mathbf{p}) \rangle \end{aligned} \tag{4.1}$$

where the factor of $(2\pi)^3$ is due to the single-baryon state normalization. The strangeness exchange scattering, i.e., the process $\overline{K}_\alpha(\mathbf{k}) + N(\mathbf{p}) \to \pi_b(\boldsymbol{\kappa}) + Y(\mathbf{q})$, is given in terms of the meson-source terms by

$$T_{\overline{K}N\to\pi Y} = i(2\pi)^3 \int d^4x\, e^{i\kappa x}\langle Y(\mathbf{q})\,|\,T(J_b^\pi(X)J_\alpha^K(0)) + \delta(x^0)[\dot\pi_b(x), J_\alpha^K(0)] - i\omega_\kappa\delta(x^0)[\pi_b(x), J_\alpha^K(0)]\,|\,N(\mathbf{p})\rangle \quad (4.2)$$

In (4.1) and (4.2) the kaon and pion source terms are defined by

$$J_\alpha^K(x) = \ddot K_\alpha + (-\nabla^2 + m_K^2)K_\alpha(x) \quad (4.3)$$

$$J_\alpha^\pi(x) = \ddot\pi_a + (-\nabla^2 + m_\pi^2)\pi_a(x) \quad (4.4)$$

Using now the commutator $i[H_K, \dot K]$ with H_K defined by (3.23) and neglecting the terms coming from the commutator $i[H_{\mathrm{Sky}}, \dot K]$ as the higher order terms, we obtain

$$\ddot K = -2i\lambda\dot K - \left[m_K^2 - \frac{m_\pi^2}{2}\left(1 - \frac{2\sigma}{F_\pi}\right) - \frac{1}{F_\pi^2}\,\partial_j\pi_a G_{ab}\partial^j\pi_b\right]K + D_jD^jK \quad (4.5)$$

$$\dot K = \Pi - i\lambda K \quad (4.6)$$

It should be noted that (4.5) is the equation of motion of K_α in the laboratory system. When we now calculate the matrix elements of the source terms between the hyperon and nucleon states, the kaon fields are transformed into fields defined in the intrinsic frame and the pion fields are reduced to the classical skyrmion fields as follows:

$$K_\alpha(x) = A_{\alpha j}\sum_N \{b_{Nj}k_N(\mathbf{r})e^{-i\tilde\omega_N t} + a_{Nj}^+k_N^c(\mathbf{r})e^{-i\omega_N t}\} \quad (4.7)$$

$$\pi_a(x) = R_{aj}(t)\cdot\tfrac{1}{2}F_\pi \mathbf{r}_{0j}\sin F(|x - \chi(t)|), \qquad \mathbf{r}_0 = \mathbf{r}/r \quad (4.8)$$

$$\sigma(x) = \tfrac{1}{2}F_\pi\cos F(|x - \chi(t)|) \quad (4.9)$$

where $F = F(r)$ is the profile function of the skyrmion, $\chi = \chi(t)$ are the translation coordinates of the center of the skyrmion, $A_{\alpha j}(t)$ are the coordinates of the $SU(2)$ isorotation, and $R_{aj}(t)$ are the coordinates of the orthogonal rotation. In (4.7) $N = \{L, T, T_3\}$, with L being the orbital angular momentum and T and T_3 being the quantum numbers of $\mathbf{T} = \mathbf{L} + \boldsymbol{\tau}/2$, and a_N and ω_N (b_N and $\tilde\omega_N$) are the annihilation operator and energy of the kaon with quantum numbers N and strangeness $S = -1$ $(+1)$, respectively. In the present paper we only consider the $S = -1$ kaon mode, where the charge-conjugate eigenmode is written as

$$k_N^c(\mathbf{r}) = k_{LT}^*(r)\begin{bmatrix} \langle T, T_3\,|\,L, T_3 + \frac{1}{2}; \frac{1}{2}, -\frac{1}{2}\rangle Y_{L,T_3+1/2}^*(\theta, \phi) \\ -\langle T, T_3\,|\,L, T_3 - \frac{1}{2}; \frac{1}{2}, +\frac{1}{2}\rangle Y_{L,T_3-1/2}^*(\theta, \phi)\end{bmatrix} \quad (4.10)$$

where $Y_{LM}(\theta, \phi)$ are the usual spherical harmonics. Using now (4.7)–(4.10), we can calculate the matrix elements of $J^K_\alpha(0)$, being a function of $\pi(0)$ and $K(0)$, between the $\langle Y|$ and $|N\rangle$ states:

$$\begin{aligned}
&\langle Y(\mathbf{q})|J^K_\alpha[\pi(0), K(0)]|N(\mathbf{p})\rangle \\
&= \frac{1}{(2\pi)^3}\int d^3\mathbf{r}\, e^{i\mathbf{k}\cdot\mathbf{r}}\langle Y|A_{\alpha n}\mathcal{J}^K_n(\mathbf{r})|N\rangle \\
&= \frac{1}{(2\pi)^3}\langle Y|A_{\alpha n}\tilde{\mathcal{J}}^K_n(\mathbf{k})|N\rangle
\end{aligned} \tag{4.11}$$

where $\mathbf{k} = \mathbf{q} - \mathbf{p}$, and

$$\tilde{\mathcal{J}}^K_n(\mathbf{k}) = \int d^3\mathbf{r}\, e^{i\mathbf{k}\cdot\mathbf{r}}\mathcal{J}^K_n(\mathbf{r}) \tag{4.12}$$

with

$$\mathcal{J}^K_n(\mathbf{r}) = \sum_N a^+_{Nn}(-\omega_N^2 - \nabla^2 + m_K^2)k^c_N(\mathbf{r}) \tag{4.13}$$

where, following Kondo *et al.* (1996), we neglect terms with $\dot{A}$ and $\ddot{A}$, as they are of higher order in the N_c^{-1} expansion, and where ω_N is the actual bound-state energy. For the positive-parity hyperons we take $L = 1$ and $T = 1/2$, and obtain

$$\tilde{\mathcal{J}}^K(\mathbf{k}) = i(\omega_\mathbf{k}^2 - \omega_1^2)\int d^3\mathbf{r}\, j_1(kr)k_1(r)\begin{bmatrix} a^+_{1/2}\sqrt{\frac{2}{3}}\,Y^*_{11} + a^+_{-1/2}\sqrt{\frac{2}{3}}\,Y^*_{10} \\ a^+_{1/2}\sqrt{\frac{1}{3}}\,Y^*_{10} + a^+_{-1/2}\sqrt{\frac{1}{3}}\,Y^*_{1-1}\end{bmatrix} \tag{4.14}$$

The hyperon states (Λ, Σ, and Σ^*), denoted by $|Y\rangle$, and nucleon state, denoted by $|N\rangle$, are defined as in Adkins *et al.* (1983) and Adkins and Nappi (1984):

$$\begin{aligned}
|Y\rangle &= |I, I_3; J, J_3\rangle \\
&= \sum_t \langle J, J_3|I, J_3 - t; \frac{1}{2}, t\rangle\sqrt{\frac{2I+1}{8\pi^2}}\,(-1)^{I+I_3}D^I_{-I_3,J_3-t}(\Theta)a^+_t\,|0\rangle
\end{aligned} \tag{4.15}$$

$$|N\rangle = |i_3, j_3\rangle = \sqrt{\frac{2}{8\pi^2}}\,(-1)^{1/2+i_3}D^{1/2}_{-i_3,j_3}(\Theta)|0\rangle \tag{4.16}$$

where Θ denotes the three Euler angles of the isospin rotation, denoted here by $A_{\alpha n} = D^{1/2}_{\alpha,n}(\Theta)$. Thus we finally obtain

$$\langle Y|A_{\alpha n}\tilde{\mathcal{J}}^K_n(\mathbf{k})|N\rangle = i\Lambda_{YN}(\boldsymbol{\sigma}\cdot\mathbf{k})\,\frac{\sqrt{4\pi}}{k}\,(\omega_\mathbf{k}^2 - \omega_1^2)\int_\epsilon^\infty dr\, r^2 j_1(kr)k_1(r) \tag{4.17}$$

The coefficients Λ_{YN} can be found in Table I. Fixing the common mass scale at m_K for kaon couplings, we obtain the pseudovector coupling constant f_{YNK}/m_K as follows:

$$\frac{f_{YNK}}{m_K} = \sqrt{4\pi}\Lambda_{YN} \lim_{\omega_k \to \omega_1} \frac{1}{k} (\omega_k^2 - \omega_1^2) \int_\epsilon^\infty dr\, r^2 j_1(kr) k_1(r) \tag{4.18}$$

since we have the pole at $\omega_k = M_Y - M_N$, that is, $\omega_k \to \omega_1$ at the leading order in the N_c^{-1} expansion. In (4.18), ϵ is the constant cutoff defined as in (2.12). The pseudoscalar coupling G_{YNK} is given by

$$G_{YNK} = \frac{M_N + M_Y}{m_K} f_{YNK} \tag{4.19}$$

The numerical results for the pseudovector coupling constants are compared to the results obtained using the complete Skyrme model (CSM) (Kondo *et al.*, 1996) and to the empirical values (Kondo *et al.*, 1996) in Table I.

From Table I we see that there is good agreement between the present results and those obtained using the complete Skyrme model (Kondo *et al.*, 1996) and the available empirical values used in Kondo *et al.* (1996).

4.2. Pion Couplings

The pion source term is derived by calculating the second time derivatives of the pion fields $\ddot{\pi}$, which are obtained from the commutator with $H_{\rm Sky}$ as follows:

$$\ddot{\pi}_a = -G_{ab}^{-1} \frac{\delta \mathcal{M}(\pi, \partial\pi)}{\delta \pi_b} + \mathcal{O}(\pi^2) \tag{4.20}$$

Table I. Coefficients Λ_{YN} and Pseudovector Coupling Constants f_{YNK}

	Λ_{YN}	$\lvert f_{YNK}/\sqrt{4\pi}\rvert$	$\lvert f_{YNK}/\sqrt{4\pi}\rvert$ (CSM) Set I[a]	Set II[a]	$\lvert f_{YNK}/\sqrt{4\pi}\rvert$ Empirical[a]
$\Lambda_{\Lambda p} \to -\Lambda_{\Lambda n}$	$1/\sqrt{2}$	1.07	1.35	0.92	0.89 ± 0.10, 0.94 ± 0.03
$\Lambda_{\Sigma^+ p} \to \Lambda_{\Sigma^- n}$	$-1/3$	0.39	0.64	0.43	—
$\Lambda_{\Sigma^0 p} \to \Lambda_{\Sigma^0 n}$	$-1/(3\sqrt{2})$	0.28	0.45	0.31	<0.43 ± 0.07, 0.25 ± 0.05
$\Lambda_{\Sigma^{*+} p} \to \Lambda_{\Sigma^{*-} n}$	$-2/\sqrt{3}$	1.81	2.21	1.50	—
$\Lambda_{\Sigma^{*0} p} \to \Lambda_{\Sigma^{*0} n}$	$-\sqrt{2}/\sqrt{3}$	1.16	1.55	1.06	—

[a] Kondo *et al.* (1996).

where π_a are replaced by the classical fields of order $N_c^{1/2}$, $\delta\mathcal{M}/\delta\pi_b = 0$ is the classical equation of motion, and the terms with $\boldsymbol{\pi}^2$ are of order $N_c^{3/2}$ and are therefore neglected.

Thus the leading source term of the pion for the positive-parity hyperons is given by

$$J_a^{\pi}(0) = (-\boldsymbol{\nabla}^2 + m_\pi^2)\pi_a(0) \tag{4.21}$$

and the pion coupling constant is written as

$$\langle Y(\mathbf{q})|\tilde{J}_a^{\pi}(\mathbf{k})|Y(\mathbf{p})\rangle = \langle Y(\mathbf{q})|R_{aj}\tilde{\mathcal{J}}_j^{\pi}(\mathbf{k})|Y(\mathbf{p})\rangle \tag{4.22}$$

where

$$\tilde{J}_a^{\pi}(\mathbf{k}) = \int d^3\mathbf{r}\; e^{i\mathbf{k}\cdot\mathbf{r}} J_a^{\pi}(\mathbf{r}), \qquad \tilde{\mathcal{J}}_j^{\pi}(\mathbf{k}) = \int d^3\mathbf{r}\; e^{i\mathbf{k}\cdot\mathbf{r}}\mathcal{J}_j^{\pi}(\mathbf{r}) \tag{4.23}$$

and the rotational matrix $R_{aj} = \frac{1}{2}\mathrm{Tr}(\tau_a \mathbf{A}\tau_j\mathbf{A}^+)$ is represented by $(-1)^a D^1_{-a,j}(\Theta)$, being consistent with $\mathbf{A}_{\alpha j} = D^{1/2}_{\alpha,j}(\Theta)$ used in the case of kaon couplings. In (4.22) we have

$$\tilde{\mathcal{J}}_j^{\pi}(\mathbf{k}) = i\,\frac{k_j}{k}\,\omega_{\mathbf{k}}^2 \int d^3r\, j_1(kr)\,\frac{F_\pi}{2}\sin F(r) \tag{4.24}$$

and

$$Y(\mathbf{q}) = Y' = \Sigma(\mathbf{q}), \qquad Y(\mathbf{p}) = Y = \Sigma(\mathbf{p}) \text{ or } \Lambda(\mathbf{p}) \tag{4.25}$$

Setting the mass scale to the pion mass m_π for the pion coupling constants, we define the pion coupling constant as

$$\frac{f_{Y'Y\pi}}{m_\pi} = 4\pi\Lambda_{Y'Y} \lim_{\omega_k\to 0} \frac{1}{k}\,\omega_{\mathbf{k}}^2 \int_\epsilon^\infty dr\, r^2 j_1(kr)\sin F(r) \tag{4.26}$$

since the Born term has the pole at $\omega_k = M_{Y'} - M_Y$, being zero at the leading order. In (4.26), similarly to (4.18), ϵ is the constant cutoff defined as in (2.12).

The numerical results for the pseudovector coupling constants are compared to the results obtained using the complete Skyrme model (CSM) (Kondo *et al.*, 1996) and to the empirical values (Kondo *et al.*, 1996) in Table II.

Table II. Coefficients $\Lambda_{Y'Y}$ and Pseudovector Coupling Constants $f_{Y'Y\pi}$

YY'	$\Lambda_{YY'}$	$\lvert f_{Y'Y\pi}/\sqrt{4\pi}\rvert$	$\lvert f_{Y'Y\pi}/\sqrt{4\pi}\rvert$ (CSM) Set I[a]	Set II[a]	$\lvert f_{Y'Y\pi}/\sqrt{4\pi}\rvert$ Empirical[a]
$\Sigma\Lambda$	1/3	0.24	0.25	0.22	0.20 ± 0.01
$\Sigma\Sigma$	1/3	0.23	0.25	0.22	0.21 ± 0.02
$\Sigma^*\Lambda$	$-1/\sqrt{3}$	0.42	0.43	0.38	0.35
$\Sigma^*\Sigma$	$1/(2\sqrt{3})$	0.21	0.21	0.19	0.19

[a] Kondo *et al.* (1996).

From Table II we see that there is a good agreement between the present results and those obtained using the complete Skyrme model (Kondo *et al.*, 1996) and the available empirical values used in Kondo *et al.* (1996). It should be noted that (see Table III in Kondo *et al.*, 1996) the empirical values for $\Sigma^*\Lambda$ and $\Sigma^*\Sigma$ are calculated using the formula for the width of the corresponding decay channel.

5. CONCLUSIONS

The present paper has shown the possibility of using the Skyrme model for the calculation of the kaon coupling constants at hyperon–nucleon vertices and the pion coupling constants at hyperon–hyperon vertices in the framework of the constant-cutoff approach to the CHK bound-state model of hyperons, without use of the Skyrme stabilizing term, being proportional to e^{-2}, which makes practical calculations more complicated and requires some low-energy approximations which otherwise are not needed to obtain relatively accurate results.

In the present paper the positive-parity hyperons such as Λ, Σ, and $\Sigma^* = \Sigma(1385)$ are the *P*-wave bound states of an antikaon and the $SU(2)$ Skyrme soliton, while $\Lambda^* = \Lambda(1405)$ is the *S*-wave bound state. Meson coupling constants are defined as matrix elements of the meson-source terms between two single-baryon states following the method developed for resolving the Yukawa coupling problem in the $SU(2)$ Skyrme soliton model (Hayashi *et al.*, 1992; Saito and Uehara, 1995).

For such a simple model, with only one arbitrary dimensional constant F_π, we have shown that the magnitudes of the meson coupling constants are found to be close to those obtained using the complete Skyrme model (Kondo *et al.*, 1996) and the phenomenological values (Lee *et al.*, 1994, 1995).

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